Faculty of Science & Technology

Mahatma Gandhi Kashi Vidyapith, Varanasi

Semester Based Syllabus (w.e.f. 2013-14)
In
M.A./M.Sc. MATHEMATICS
Scheme of Examination

Semester	Paper	Marks
First	I-MAT 101 – Algebra - I	100
	II- MAT 102 – Real Analysis - I	100
	III- MAT 103 – Basic Topology	100
	IV- MAT 104 – Complex Analysis	100
	V- MAT 105 – Hydrodynamics	100
Second	I-MAT 201 – Algebra - II	100
	II- MAT 202 – Measure and Integration	100
	III- MAT 203 – Classical Mechanics	100
	IV- MAT 204 – Mathematical Methods	100
	V- MAT 205 - Special Theory of Relativity	100
Third	I-MAT 301 - Topology	100
	II- MAT 302 – Advanced Linear Algebra	100
	III- MAT 303– Partial differential equations & Integral Equations	100
	Elective (Optional) Papers (Any two of the following)	
	IV& V - MAT 304 – Differential Geometry of Manifolds – I	100
	MAT 305 Operations Research – I	100
	MAT 306 General Relativity and Cosmology	100
	MAT 307 Advanced Discrete Mathematics	100
Fourth	I- MAT 401 – Functional Analysis	100
	II- MAT 402 – Normed Linear Spaces and Theory of Integration	100
	Elective (Optional) Papers (Any two of the following)	
	III & IV- MAT 403 – Differential Geometry of Manifolds-II	100
	MAT 404 – Fluid Mechanics	100
	MAT 405 – Algebraic Topology	100
	MAT 406 – Operations Research - II	100
	V – Viva – Voce (Based on Theory Papers)	100

SEMESTER- I

MAT101 ALGEBRA - I

UNIT I: Action of a group *G* on a set *S*, Equivalent formulation as a homomorphism of *G* to *T*(*S*), Examples, Stabilizer (Isotropy) subgroups and Orbit decomposition, Class equation of an action, Its particular cases (left multiplication and conjugation), Conjugacy class equation, Transitive and effective actions, Equivalence of actions, Core of a subgroup.

UNIT II: Subnormal and normal series, Zassenhaus's lemma, Schreier's refinement theorem, Composition series, Jordan-Hölder's theorem, Chain conditions, Examples, Internal and External direct products and their relationship, Indecomposability. Sylow subgroups, Sylow's Theorem I, II and III, *p*-groups, Examples and applications, Groups of order *pq*.

UNIT III: Commutators, Solvable groups, Solvability of subgroups, factor groups and of finite *p*-groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

UNIT IV: Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Divisor chain condition, Unique factorization domains, Examples and counter examples, Chinese remainder theorem for rings and PID's, Polynomial rings over domains, Eisenstein's irreducibility criterion, Unique factorization in polynomial rings over UFD's.

Books Recommended:

1. D. S. Dummit and R.M. Foote, Abstract Algebra, John Wiley, N.Y, 2003.
2- N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3- N. Jacobson, Basic Algebra, Vol. I, Hindustan Publishing Co., New Delhi, 1984.
4- RamjiLal, Algebra, Vols. I & II, Shail Publications, Allahabad, 2002.

MAT 102 Real Analysis-I

UNIT I: Definition and existence of Riemann-Stieltjes integral, Conditions for R-S integrability. Properties of the R-S integral, R-S integrability of functions of a function.Integration and differentiaion, Fundamental theorem of Calculus.

Unit II: Series of arbitrary terms. Convergence, divergence and oscillation, Absolute Convergence, Abel's and Dirichilet's tests.Multiplication of series.Rearrangements of terms of a series, Riemann's theorem and sum of series, Sequences and series of functions.

Unit III: Pointwise and uniform convergence, Cauchy's criterion for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjies integration, uniform convergence and differentiation. Weierstrass approximation theorem.Power series.Uniqueness theorem for power series, Abel's and Tauber's theorems.

Unit IV: Function of Several Variables, Linear transformations, Derivatives in an open subset of , Jacobian matrix and Jacobians, Chain rule and its matrix form, Interchange of order of differentiation, Derivatives of higher ordersTaylor's theorem, Inverse function theorem, Implicit function theorem, Extremum problems with constraints, Lagranges multiplier method.

Books Recommended:

1- Walter Rudin, *Principle of Mathematical Analysis* (3rd edition) McGraw- Hill Kogakusha, 1976, International Student Edition.
2- T. M. Apostol, *Mathematical Analysis*, Narosa Publishing House, New Delhi, 1985.
3- S Lang, Analysis I and II, Addision-Wesley Pub. Co. 1969

MAT 103 : BasicTopology

UNIT I: Definition and Examples of Metric spaces, Equivalent metrics, characterization of open sets in terms of open sphere, characterization of closed sets in terms of closed spheres, Countability of metric space, Continuity of functions, Properties of continuous functions , Homeomorphisms.

UNIT II: Connectedness in metric spaces, Connected sets in the real line, Continuity and connectedness, Compactness, closed subset of a compact space, compact subset of a metric space, Continuity and compactness.

Unit III: Definition and examples of topological spaces. Closed sets.Closure. Dense sets. neighborhoods, interior, exterior, and boundary. Accumulation points and derived sets.Bases and sub-bases.Subspaces and relative topology. Alternative methods of defining a topology in terms of Kuratowski closure operator and neighborhood systems.

Unit IV: Continuous functions and homeomorphism. First and second countable space.Lindelöf spaces.Separable spaces.The separation axioms *T0*, *T1*, *T2*, *T3½*, *T4*; their characterizations and basic properties.Urysohn's lemma.Tietze extension theorem.

Books Recommended:

1. J. L. Kelley, *General Topology*, Van Nostrand, 1995.
2. K. D. Joshi, *Introduction to General Topology*, Wiley Eastern, 1983.
3. James R. Munkres, *Topology,* 2nd Edition, Pearson International, 2000.
4. J. Dugundji, *Topology*, Prentice-Hall of India, 1966.

MAT 104 : COMPLEX ANALYSIS

UNIT I: Schwarz lemma, Analytic Automorphisms of the Disc, The Upper Half Plane, Other Examples, Schwarz reflection, Reflection across analytic arcs, application of Schwarz reflection.

UNIT II: (The Riemann Mapping Theorem) Statement of the Theorem, Compact Sets in Function Spaces, Proof of the Riemann Mapping Theorem, Behavior at the Boundary.

UNIT III: Analytic continuation Along a Curve, The Dilogarithm, Application to Picard's theorem, Jensen's Formula, The Picard-Borel Theorem, Bounds by the Real Part, Borel-Caratheodory Theorem.

UNIT IV: Entire Functions with Rational Values, ThePhragmen-Lindelof and Hadamard Theorems, Infinite Products, Weierstrass Products, Weierstrass Products, Meromorphic Functions, Mittag-Leffler Theorem.

Books Recommended:

1. Serge lang, Complex Analysis, Fourth edition, Springer. (Chapters vii, ix-xii).
2. J. Bak and D. J. Newman, Complex Analysis, Springer.
3. J. B. Conway, Complex Analysis, Springer.

MAT 105 : Hydrodynamics

UNIT I: Equation of continuity, Boundary surfaces, streamlines, Velocity potential, Irrotational and rotational motions, Vortex lines, Euler's Equation of motion, Bernoulli's theorem, Impulsive actions.

UNIT II: Motion in two-dimensions, Conjugate functions, Source, sink, doublets and their images, conformal mapping, Circle Theorem,

UNIT III: Two-dimensional irrotational motion produced by the motion of circular cylinder in an infinite mass of liquid, Theorem of Blasius, Motion of Eliptic Cylinder.

UNIT IV: Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere.Concentric Spheres.

Books Recommended:

1. W. H. Besant and A. S. Ramsey, *A Treatise on Hydrodynamics*, CBS Pub. Delhi, 1988.
2. S. W. Yuan, *Foundations of Fluid Dynamics*, Prentice-Hall of India, 1988.

SEMESTER- II

MAT 201 : ALGEBRA- II

UNIT I: Modules over a ring, Endomorphism ring of an abelian group, *R*-Module structure on an abelian group *M* as a ring homomorphism from *R* to EndZ (*M*), submodules, Direct summands, Annihilators, Faithful modules, Homomorphism, Factor modules, Correspondence theorem, Isomorphism theorems, HomR (*M*, *N*) as an abelian group and HomR (*M*, *M*) as a ring, Exact sequences, Five lemma, Products, coproducts and their universal property, External and internal direct sums.

UNIT II: Free modules, Homomorphism extension property, Equivalent characterization as a direct sum of copies of the underlying ring, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization, Divisible groups, Examples of injective modules, Existence of enough injectives.

UNIT III: Noetherian modules and rings, Equivalent characterizations, Submodules and factors of noetherian modules, Characteristic of a field, Prime subfields, Field extensions, Finite extensions, Simple extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields, Splitting fields and their uniqueness.

UNIT IV: Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability, Automorphisms of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, normal closures, Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, N.Y., 1974.
3. I. A. Adamson, An Introduction to Field Theory. Oliver & Boyd, Edinburgh, 1964.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.
5. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
6. RamjiLal, Algebra, Vol. 2, Shail Publishing House, Allahabad, 2002

MAT 202: MEASURE AND INTEGRATION

UNIT I: Semi-algebras, algebras, monotone class, σ -algebras, measure and outer measures, Caratheödory extension process of extending a measure on a semi-algebra to generated σ -algebra, completion of a measure space.

UNIT II: Borel sets, Lebesgue outer measure and Lebesgue measure on *R*, translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets, the Cantor-Lebesgue function.

UNIT III: Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, simple functions and their integrals, Lebesgue integral on *R* and its properties, Riemann and Lebesgue integrals.

UNIT IV: Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem, Minkowski's and Hölder's inequalities.

Books Recommended:

1- H. L. Royden and P. M. Fitzpatrick, Real Analysis, (Fourth edition), P. H.l, 2010.
2- P. R. Halmos, Measure Theory, Grand Text Mathematics, 14, Springer, 1994.
3- I. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, New Delhi, 2005.
4- E. Hewit and K. Stromberg, Real and Abstract Analysis, Springer, 1975.

MAT 203 : CLASSICAL MECHANICS

UNIT I: The linear momentum and the angular momentum of a rigid body in terms of inertia constants, kinetic energy of a rigid body, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes. Euler's equations of motion, motion under no forces, the invariable line and the invariable cone, the theorems of Poinsot and Sylvester, Eulerian angles and the geometrical equations of Euler.

UNIT II: Generalized co-ordinates, geometrical equations, holonomic and non-holonomic systems, configuration Space, Lagrange's equations using D' Alembert's Principle for a holonomic conservative system, deduction of equation of energy when the geometrical equations do not contain time *t* explicitly, Lagrange's multipliers case, deduction of Euler's dynamical equations from Lagrange's equations.

UNIT III: Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints, stationary property of normal modes, Lagrange equations for impulsive motion.

UNIT IV: Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, phase space and Hamilton's Variational principle, the principle of least action, canonical transformations, Hamilton-Jacobi theory, Integrals of Hamilton's equations and Poisson- Brackets, Poisson-Jacobi identity.

Books Recommended:

1. A. S. Ramsey, Dynamics, Part II, CBS Publishers & Distributors, Delhi, 1985.
2. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company, London, 1969.
3. K. C. RANA AND P. C. JOAG, Classical Mechanics, Narosa.Pub.

MAT 204: MATHEMATICAL METHODS

UNIT I: (Fourier Series)

Periodic functions, Trigonometric series, Fourier series, Euler formulas, Functions having arbitrary periods, Even and Odd functions, Half-range expansions, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error.

UNIT II: (Boundary-value problems and Transforms) Orthogonal and Orthonormal sets of functions, Generalized Fourier series, Sturm- Liouville problems, Examples of Boundary-value problems which are not Sturm- Liouville problems, Definition, Existence and Linearity of Laplace Transform.

UNIT III: (Fourier Transform) Fourier Integrals, Fourier Cosine and Sine Integrals, Inverse Fourier Transform, Fourier Cosine and Sine Transform, Complex form of the Fourier Transform, Linearity of the Fourier Transform.

UNIT IV: Calculus of Variations: Functionals and extremals, Variation and its properties, Euler equations, Cases of several dependent and independent variables, Functionals dependent on higher derivatives, Parametric forms, Simple applications.

Books Recommended:

1. E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8^{th} Edition, 2001.
2. L. Elsgolts, Differential Equations and Calculus of Variations, Mir Publishers, 1970.
3. A. S. Gupta, Calculus of Variations, Prentice Hall of India, New Delhi, 1999.
4. J. H. Davis, Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004.

MAT 205: SPECIAL THEORY OF RELATIVITY

UNIT I: Review of Newtonian Mechanics, Inertial frame, Speed of light and Galilean relativity, Michelson-Morley experiment, Lorentz-Fitzerold contraction hypothesis, relative character of space and time, postulates of special theory of relativity, Lorentz transformation equations and geometrical interpretation, Group properties of Lorentz transformations.

UNIT II: Relativistic kinematics, composition of parallel velocities, length contraction, time dialation, transformation equations, equations for components of velocitiy and acceleration of a particle and contraction factor.

UNIT III: Geometrical representation of of space time, four dimensional minkowskian space of special relativity, time-like intervals, light-like and space- like intervals, Null cone, proper time, world line of a particle, four vectors and tensors in Minkowskian space time.

UNIT IV: Relativistic mechanics- Varriations of mass with velocity, equivalence of mass energy, transformation equation for mass, momentum and energy, Energy momentum for light vector, relativistic force and transformation equation for its components, relativistic Lagrangian and Hamiltonian, relativistic equations of motion of a particle, energy momentum tensor of a continuous material distribution.

Books Recommended:

1. C. Mollar, Theory of relativity, Clarendon press, 1952.
2. R. Resnick, Introduction to special relativity, Wiley Eastern Pvt. Ltd. 1972.
3. J L Anderson, Principles of relativity, Academic Press.

SEMESTER- III

MAT 301: TOPOLOGY

Unit I: Compactness. Basic properties of compactness.Compactness and finite intersection property.Sequential, countable, and B-W compactness.Local compactness. One-point compactification. Connected spaces and their basic properties.Components.Locally connected spaces.Continuity and connectedness.

Unit II: Tychonoff product topology in terms of standard sub-base and its characterizations.Product topology and separation axioms, connected-ness, and compactness (incl. the Tychonoff's theorem), product spaces.

UNIT III: Homotopy of paths, the fundamental group, covering spaces, fundamental group of circle, punctured plane, n-sphere, figure 8 and of surfaces.

Unit IV: Essential and Inessential maps, equivalent conditions, Fundamental theorem of algebra, Vector fields and fixed points, Brouwer fixed point theorem for disc, Homotopy type and Jordan separation Theorem.

Books Recommended:

1. J. L. Kelley, *General Topology*, Van Nostrand, 1995.
2. K. D. Joshi, *Introduction to General Topology*, Wiley Eastern, 1983.
3. James R. Munkres, *Topology,* 2nd Edition, Pearson International, 2000.
4. J. Dugundji, *Topology*, Prentice-Hall of India, 1966.

MAT302: ADVANCED LINEAR ALGEBRA

UNIT I: Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, T-conductors and T-annihilators, Minimal polynomials of linear operators and matrices, Characterization of diagonalizability in terms of multiplicities and also in terms of the minimal polynomial, Triangulability, Simultaneous triangulation and diagonalization.

UNIT II: Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into $T(M)$ and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

UNIT III: Reduction of matrices over polynomial rings over a field, Similarity of matrices and $F[x]$-module structure, Projections, Invariant direct sums, Characterization of diagonalizability in terms of projections, Primary decomposition theorem.

UNIT IV: Diagonalizable and nilpotent parts of a linear operator, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Semisimple operators, Taylor formula.

Books Recommended:

1. K. Hofmann and R. Kunze, Linear Algebra. Prentice Hall of India, New Delhi, 1972.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y. 2003.
3. H. Helson, Linear Algebra, Hindustan Book Agency, New Delhi, 1994.
4. N. Jacobson, Basic Algebra, Vol. 1, Hindustan Publishing Co., New Delhi, 1984.
5. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
6. T. W. Hungerford, Algebra, Springer (India), Pvt. Ltd., 2004.
7. C. Musili, Rings and Modules, Narosa Publishing House, New Delhi, 1994

MAT 303: PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

UNITI: Formation of P.D.E.'s, First order P.D.E.'s, Classification of first order P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi - linear equations, Integral surfaces through a given curve, Orthogonal surfaces to a given system of surfaces, Characteristic curves.

UNIT II: Pfaffian differential equations, Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.'s.

UNIT III: Method of separation of variables: Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations in a string of finite length and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

UNIT III: Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books Recommended:

1- I. N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1957.
2- T. Amaranath, An Elementary Course in Partial Differential Equations, Narosa Pub. New Delhi, R. P. Kanwal, Linear Integral Equations, Birkhäuser, Inc., Boston, MA, 1997.

Elective (Optional) papers (Any two of the following)

MAT 304 : DIFFERENTIAL GEOMETRY OF MANIFOLDS I

UNIT I: Definition and examples of differentiable manifolds. Tangent spaces.Jacobian map and parameter group of transformations, Lie derivatives. Immersions and imbeddings.

UNIT II: Exterior algebra. Exterior derivative, Lie groups and lie algebras. Product of two Lie groups.One parameter subgroups and exponential maps.Examples of Lie groups.

UNIT III: Homomorphism and isomorphism. Lie transformation groups. General linear groups.

UNIT IV: Principle fibre bundle, linear frame bundle, associated fibre bundle, vector bundle. Tangent bundle.Induced bundle. Bundle homomorphism.

Books Recommended

1. R.S. Mishra, A course in tensors with applications to Riemannian Geometry Pothishala (Pvt.) Ltd., 1965.
2. R.S. Mishra, Structures on a differentiable manifold and their applications. ChandramaPrakashan, Allahabad, 1984.
3. B.B. Sinha, An introduction to modern differential geometry, Kalyani Publishers, New Delhi, 1982.

MAT 305: Operations Research I

UNIT I: Operations Research and its Scope. Necessity of Operations Research in Industry.Linear Programming – Simplex Method.

UNIT II: Duality and Sensitivity Analysis. Dual Simplex Method.Parametric Linear Programming.Upper Bound Technique.

UNIT III: Transportation and Assignment Problems.

UNIT IV: Integer programming. Dynamic Programming.

Books Recommended:

1. H.A. Taha. Operations Research – An Introduction, Macmillan Publishing Co., Inc., New York.
2. S.S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.

MAT 306: GENERAL RELATIVITY AND COSMOLOGY

UNIT I: Review of special theory of relativity and the Newtonian Theory of Gravitation. Principle of equivalence and general covariance.Geodesic Principle.Newtonian approximation.

UNIT II: Schwarzchild external solution and its isotropic form. Planetary orbits and analogues of Kepler's law in genral relativity.Advance or perihelion of a planet.Bending of light rays in gravitational field.Gravitational redshift of spectral lines.

UNIT III: Energy momentum tensor of a perfect fluid. Schwarzchild internal solution.Boundary conditions.Energy momentum tensor of an electromagnetic field.Einstein-Maxwell equations.Reissner-Nordstrom solution.

UNIT IV: Mach's Principle. Einstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe.Hubble's law.Cosmological principle's Wey'Is postulate.Derivation of Robertson-Walker metric.

References :

1. C.E. WeatherburnAn Introduction ToReimanian Geometry and the tensor Calculus, Cambridge University Press, 1950.
2. J.V. Narlikar, General Relativity and Cosmology, The Macmillan Company of India Ltd., 1978.

MAT 307: Advanced Discrete Mathematics

UNIT I: Lattices – Lattices as partially ordered sets. Their properties.Lattices as Algebraic systems.Sublattices. Direct product, and Homomorphisms. Some Special Lattices, e.g., Complete, Complemented and Distributive Lattices.

UNIT II: Boolen Algebras – Boolean Algebras as Lattices. Various Boolean identities.Direct Product and Homomorphisms.Cannonical Forms.Minimization of Boolean Functions. Application of Boolean Algebra to Switching Theory (using AND, OR & NOT gates). The Karnaughj Map method.

UNIT III: Graph Theory – Definition of (undirected) Graphs. Paths, Circuits, Cycles and Subgraphs, Induced Subgraphs. Degree of vertex.Connectivity, Planar Graphs and their properties.Tress.Euler's Formula for connected Planar Graphs. Complete & Complete Bipartite Graphs.

UNIT IV: Spanning Tress. Cut-sets, Fundamental Cut-sets and Cycles.Minimal Spanning Tress and Kruskal's Algorithm.Matrix Representation of Graphs.Euler's Theory of a Vertex.Weighted undirected Graphs.Strong Connectivity &Warshall's Algorithm.Directed Tress. Search Trees. Tree Traversals.

SEMESTER- IV

MAT 401: FUNCTIONAL ANALYSIS

UNIT I: Normed linear spaces, Examples and its topological properties, Banach spaces, Continuous linear transformations, Spaces of continuous linear transformations from a linear space to a Banach space, Continuous linear functional.

UNIT II: Hahn-Banach Theorem, Open mapping theorem, Closed graph theorem, Banach-Steinhaus theorem, Uniform boundedness principle.

UNIT III: Hilbert Spaces, Schwarz's inequality, orthogonal complement of a subspace, orthonormal bases, Continuous linear functionals on Hilbert spaces, Riesz Representation Theorem, Reflexivity of Hilbert Spaces, Unitary operators on a Hilbert space, self-adjoint and normal operators, adjoint of an operator on a Hilbert space, projections of Hilbert spaces.

UNIT IV: Determinant and the Spectrum of an operator, Spectral Theorem.

Books Recommended:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. S. Ponnusamy, Foundations of Functional Analysis, Narosa Publishing House, New Delhi, 2002.

MAT 402: Normed Linear Spaces and Theory of Integration

UNIT I: The L L^p-space. Convex functions. Jensen's inequality, Holder and Minkowski inequalities, Completeness of L^p. Convergence in measure, Almost uniform convergence.

UNIT II: Signed measure. Hahn and Jordan decomposition theorems.Absolutely continuous and singular measures.Radon Nikodyn theorem.Labesgue decomposition.Riesz representation theorem, Extension theorem (carathedory).

UNIT III: Product measures, Fubini's theorem, Baire sets, Baire measure, Continuous functions with compact support.

UNIT IV: Regularity of measures on locally compact spaces. Integration of continuous functions with compact support.Reisz-Markoff theorem.

Books Recommended

1. H. L. Royden, *Real Analysis*, Macmillan, 4th Edition, 1993.
2. P. R. Halmos, *Measure Theory*, Van Nostrand, 1950.
3. S. K. Berberian, *Measure and Integration*, Wiley Eastern, 1981.
4. A. E. Taylor, *Introduction to Functional Analysis*, John Wiley, 1958.
5. G. de Barra, *Measure Theory and Integration*, Wiley Eastern, 1981.
6. R. G. Bartle, The Elements of Integration, John Wiley, 1966.
7. Inder K. Rana, *An Introduction to Measure and Integration*, Narosa Publishing House, 1997.

Elective (Optional) Papers (Any two of the following)

MAT 403: DIFFERENTIAL GEOMETRY OF MANIFOLDS II

UNIT I: Riemannian manifolds, Riemannian connection, Curvature tensor, Sectional Curvature.

UNIT II: Shur's theorem, Geodesics in a Riemnnian manifold. Projective curvature tensor, Conformal curvature tensor.

UNIT III: Submanifolds and hyper surfaces. Normals, Gauss' formulas.Weingarten equations.Lines of Curvatures.

UNIT IV: Generalized Gauss and Minardi-Codazzi equations. Almost complex manifolds, Nijenhuis tensor.Contravariant and covariant almost analytic vector fields.F Connection.

Books Recommended:

1. R.S. Mishra, A course in tensors with applications to Riemannian Geometry Pothishala (Pvt.) Ltd., 1965.
2. R.S. Mishra, Structures on a differentiable manifold and their applications. ChandramaPrakashan, Allahabad, 1984.
3. B.B. Sinha, An introduction to modern differential geometry, Kalyani Publishers, New Delhi, 1982.

MAT 404: FLUID MECHANICS

UNIT I: Elementary notions of fluid motion: Body forces and surface forces, Nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newton's law of viscosity, Navier- Stokes equation (sketch of proof).

UNIT II: Equation of motion for inviscid fluid, Energy equation, Vortex motion- Helmholtz's vorticity theorem and vorticity equation, Kelvin's circulation Theorem, Mean Potential over a spherical surface, Kelvin's Minimum kinetic energy Theorem, Acyclic irrotational motion.

UNIT III: Wave motion in a gas. Speed of Sound.Equation of motion of a gas.Subsonic, Sonic and Supersonic flows of a gas. Isentropic gas flows.

UNIT IV: Normal and oblique shocks. Plane Poiseuille and Couette flows between two parallel plates. Unsteady flow over a flat plate.Reynold's number.

Books Recommended:

1. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. Van Nost.CompLondon, 1968.
3. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
4. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
5. F. Chalton. A text book of fluid dynamics. CBS Publication, New Delhi.

MAT 405: ALGEBRAIC TOPOLOGY

UNIT I: Attaching spaces, Spheres, real and complex projective spaces and generalized torus as attaching spaces, Hopf map, CW-complexes and cellular maps.

UNIT II: Homotopic maps, relative homotopy, homotopy type, space of paths and loops, fundamental group, simply connected space, calculation of fundamental group of the circle, fundamental group of product of spaces, contractible spaces, example of a space having non-abelianhomotopy group, inessential maps.

UNIT III: Singular simplices and complexes, singular homology groups, relative homology groups, verification of Eilenberg-Steenrod axioms, Mayer-Vietories sequence, homology of complexes, computation of homology groups of spheres, torus, real and complex projective spaces, relation between homotopy and homology groups, Fundamental Theorem of Algebra.

UNIT IV: Compact-open topology, exponential law, higher homotopy groups of a space, homotopy exact sequence of a pair of spaces (sketch of the proof), Poincaré- Hurewicz Theorem.

Books Recommended:

1. J. J. Rotman, An Introduction to Algebraic Topology, Springer-Verlag, 1988.
2. E. H. Spanier, Algebraic Topology, McGraw Hill, 1966.
3. W. S. Massey, Algebraic Topology-An Introduction, Springer-Verlag, 1988.
4. G. E. Bredon, Topology and Geometry, Springer-Verlag, New York, 1993.

MAT 406 : OPERATIONS RESEARCH II

UNIT I: Network Analysis – Shortest path problem. Minimum Spanning Problem.Maximum Flow Problem. Minimum Cost Flow Problem. Network Simplex Method. Project Planning and Control with PERT – CPM

UNIT II: Game Theory – Two Person, Zero – Sum Games. Games with Mixed Strategies.Graphic Solution.Solution by Linear Programming.

UNIT III: Nonlinear Programming – One and Multi-Variable Unconstrained Optimization. Kuhn-Tucker.Conditions for constrained Optimization.Quadract Programming. Separable Programming.

UNIT IV: Convex Programming. Non-Convex Programming. Linear Goal Programming

Books Recommended

1. H.A. Taha. Operations Research – An Introduction, Macmillan Publishing Co. Inc., New York.
2. S.S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.